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Scan the QR code for answers to all exercise and Practice Test questions.

# Chapter 1

## Numbers

### Different types of number

#### Rational numbers

A **rational number** is a number which can be expressed as a fraction of the form  $\frac{a}{b}$ , where  $a$  and  $b$  are both whole numbers. Rational numbers consist of:

1. All **natural numbers** e.g. 1, 2, 3, 4, ...
2. All **integers** (positive, negative and zero) e.g. ..., -3, -2, -1, 0, 1, 2, 3, ...
3. All **fractions** (mixed numbers, proper and improper fractions) e.g.  $1\frac{3}{7}$ ,  $\frac{1}{2}$ ,  $\frac{4}{3}$ , ...
4. All **recurring decimals** e.g. 0.666666..., 0.23232323..., 2.813813813, ...
5. All **terminating decimals** e.g. 0.12, 1.278, 14.6, ...

#### *What's more*

Recurring decimals are also called **repeating** decimals.

## Irrational numbers

An **irrational number** is a number which cannot be expressed as a fraction. Here are some common examples of irrational numbers:

$$\sqrt{2}, \sqrt{3}, \pi, \sqrt[3]{7}, \dots$$

## Real numbers

The **real numbers** are all the rational and irrational numbers.

## Square numbers

A **square number** is a positive integer which is the square of another integer. The first few square numbers are 1, 4, 9, 16, 25 and 36;  $1 = 1^2$ ,  $4 = 2^2$ ,  $9 = 3^2$ ,  $16 = 4^2$ , etc.

## Factors

A **factor** of an integer (whole number) is an integer that divides it exactly.  
e.g. The positive factors of 6 are 1, 2, 3 and 6.

## Multiples

A **multiple** of an integer is that number multiplied by another integer.  
e.g. The first four multiples of 6 are 6 ( $6 \times 1$ ), 12 ( $6 \times 2$ ), 18 ( $6 \times 3$ ), and 24 ( $6 \times 4$ ).

### Example 1.1

Which of the following are square numbers: 1, 2, 6, 9, 64?

*Solution*

$$1 = 1^2, 9 = 3^2, 64 = 8^2$$

So the square numbers are 1, 9 and 64.

### Example 1.2

From the list of numbers:  $\sqrt{3}$ , 3.14,  $\pi$ ,  $\sqrt{4}$ ,  $\frac{103}{7}$ ,  $\sqrt[3]{7}$ , write down:

- a all the integers,
- b all the rational numbers,
- c all the irrational numbers.

*Solution*

a  $\sqrt{4}$  (as  $\sqrt{4} = 2$ )

b 3.14,  $\sqrt{4}$ ,  $\frac{103}{7}$  (3.14 is rational as  $3.14 = \frac{314}{100}$ )

c  $\sqrt{3}$ ,  $\pi$ ,  $\sqrt[3]{7}$

## What's more

Positive integers with three or more factors are called **composite** numbers. Every positive integer, except for 1, is either prime or composite.

## Prime numbers

A **prime number** is a positive integer greater than 1 that is not divisible by any integer except 1 and itself. This definition means *the number 1 is not counted as a prime*.

Below is a list of all the prime numbers between 1 and 50:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

The **prime factors** of a number are the prime numbers which divide that number exactly. For example, 2 and 3 are the prime factors of 6.

### Example 1.3

Identify the prime numbers in each of the following sets of numbers:

- a** 3, 9, 17, 21, 15  
**b** 8, 14, 17, 23, 27  
**c** 7, 24, 13, 47, 57

### Solution

- a** 3, 17  
**b** 17, 23  
**c** 7, 13, 47

A quick way to find the prime factors of a number is by **short division**. Example 1.4 shows you how to do this.

### Example 1.4

Find the prime factors of the following numbers by short division and write the numbers as products of primes:

- a** 12    **b** 84    **c** 124

### Solution

$$\begin{array}{r} \mathbf{a} \quad 2 \overline{)12} \\ \quad 2 \overline{)6} \\ \quad \quad 3 \end{array}$$

We stop as 3 is a prime

The prime factors are 2 and 3. Also,  $12 = 2^2 \times 3$ .

$$\begin{array}{r} \mathbf{b} \quad 2 \overline{)84} \\ \quad 2 \overline{)42} \\ \quad \quad 3 \overline{)21} \\ \quad \quad \quad 7 \end{array}$$

7 is prime so we stop here

The prime factors are 2, 3 and 7. Also,  $84 = 2^2 \times 3 \times 7$ .

$$\begin{array}{r} \text{c} \quad 2 \overline{)124} \\ \quad 2 \overline{)62} \\ \quad \quad 31 \end{array}$$

31 is a prime

The prime factors are 2 and 31. Also,  $124 = 2^2 \times 31$ .

### Highest common factor (HCF)

The **highest common factor (HCF)** of two integers  $a$  and  $b$  is the largest integer that divides both  $a$  and  $b$  without a remainder.

For example:  $\text{HCF}(8, 12) = 4$  because 4 divides both 8 and 12 and 4 is also the largest integer that does this.  $\text{HCF}(15, 30) = 15$  because 15 is the largest integer that divides both 15 and 30.

### Lowest common multiple (LCM)

The **lowest common multiple (LCM)** of two integers  $a$  and  $b$  is the smallest integer that is a multiple of both  $a$  and  $b$ .

For example:  $\text{LCM}(8, 12) = 24$  because both 8 and 12 divide 24, and there is no integer smaller than 24 that 8 and 12 both divide.

The HCF and LCM can again be found by short division. Example 1.5 shows you how to do this.

### Example 1.5

Find the HCF and the LCM of:

- a 30 and 42
- b 60 and 100
- c 24, 36 and 60

### Solution

$$\begin{array}{r} \text{a} \quad 2 \overline{)30 \quad 42} \\ \quad 3 \overline{)15 \quad 21} \\ \quad \quad 5 \quad 7 \end{array}$$

5 and 7 have no common factors, except 1, so we stop.

HCF of 30 and 42 =  $2 \times 3 = 6$

LCM of 30 and 42 =  $2 \times 3 \times 5 \times 7 = 210$

$$\begin{array}{r} \text{b} \quad 2 \overline{)60 \quad 100} \\ \quad 2 \overline{)30 \quad 50} \\ \quad \quad 5 \overline{)15 \quad 25} \\ \quad \quad \quad 3 \quad 5 \end{array}$$

HCF of 60 and 100 =  $2 \times 2 \times 5 = 20$

LCM of 60 and 100 =  $2 \times 2 \times 5 \times 3 \times 5 = 300$

### What's more

The HCF is sometimes called the GCD (short for 'greatest common divisor').



- 11** There are 900 litres of liquid chemical in a large tank, correct to the nearest 10 litres. The chemical is to be poured into a number of small tanks, each of a capacity of 3.5 litres, correct to the nearest 0.1 litres. Find the maximum possible number of small tanks required.
- 12** The side length of a square is measured to be 6.81 cm correct to 3 significant figures.
- Find the lower bound and upper bound of the perimeter of the square.
  - Using a suitable level of accuracy, find the perimeter of the square.
- 13** The weight of a parcel is measured as 26 kg correct to the nearest kg.
- Find the lowest possible weight of the parcel.
  - If 5 identical parcels are weighed together, find the upper bound of the total weight.
- 14** The weights of Ben and Michael are 65 kg and 58 kg respectively, correct to the nearest kg.
- Find the upper bound of the difference between their weights.
  - Find the lower bound of the difference between their weights.
- 15** In a physics lesson, a student uses the formula  $T = 2\pi\sqrt{\frac{l}{g}}$  to calculate  $T$ .
- It is given that  $l = 3.56$ ,  $g = 9.81$  and  $\pi = 3.14$ , all correct to 3 significant figures.
- Find the lower bound of the value of  $T$ .
  - Find the upper bound of the value of  $T$ .
- 16** The side length of a cube is 7.4 cm, correct to 2 significant figures. Find the difference between the upper bound and the lower bound of the total surface area of the cube.

## Summary

### Types of number

- **Natural numbers:** 1, 2, 3, 4, .....
- **Integers:** -6, -3, 0, 4, 12, 300
- **Fractions:**  $\frac{1}{2}$ ,  $-\frac{2}{5}$ ,  $\frac{5}{4}$ ,  $\frac{8}{2}$ ,  $-\frac{100}{10}$
- **Recurring decimals:**  $0.3333\dots (= 0.\dot{3})$ ,  $0.121212\dots (= 0.1\dot{2})$
- **Terminating decimals:** 5.6, 0.04, -2.781
- **Irrational numbers:**  $\pi$ ,  $\sqrt{2}$ ,  $-\sqrt{7}$